Introduction to Convection

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Mechanical Engineering 375

Heat Transfer

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California State University Northridge

Outline

- Quiz five results and comments
- New topic: how to compute h
- Basic heat transfer coefficient
- Use of dimensionless parameters
- Classification of flows
- Flow properties
- Boundary layer
- Analytical equations

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Quiz Five Grades

- 22 Students
- 25 Maximum possible
- Mean (average) = 20.1
- Standard deviation = 3.78
- Median = 21
- Grade distribution

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   13 15 15 15 16 16 18
 19 20 20 21 21 21 21 22
   23 24 24 24 25 25  25
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Quiz Five Comments

- Highest quiz average I have ever seen!
- Minor problems
  - Convert c_p from kJ/kg▪°C to J/kg▪°C
  - Note that V/A used in lumped parameter Biot modulus is not same as length used in calculations with charts
  - Compute $\alpha = k/\rho c_p$
  - Chart solution is more accurate because Bi is too large to use lumped parameter

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Review Convection Basics

This example is an external flow (flow over an object) with velocity relative to the object

The reference velocity, V∞, and temperature T∞ are called the free-stream values (far from the object)

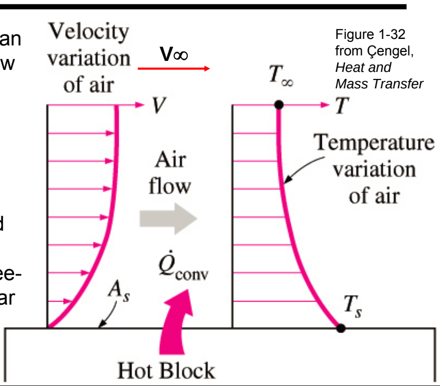

Figure 1-32 from Çengel, *Heat and Mass Transfer*

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Review Convection Basics II

V∞ = reference velocity

T_∞ = reference temperature

A_s = surface area

T_s = surface temperature

V, T = variable velocity and temperature

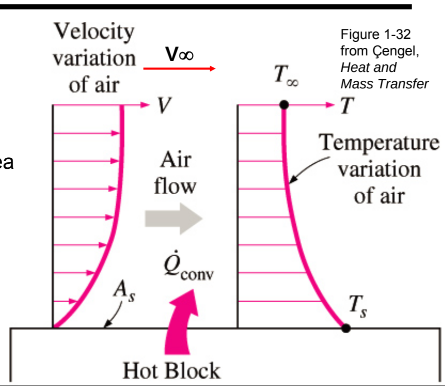

Figure 1-32 from Çengel, *Heat and Mass Transfer*

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Review Convection Basics III

$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$

h = heat transfer coefficient (W/m^2·K) or Btu/hr·ft^2·°F

h is found from empirical or theoretical equation

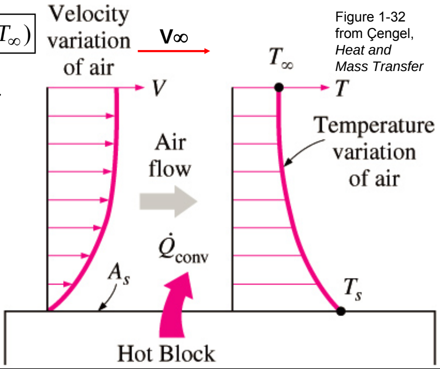

Figure 1-32 from Çengel, *Heat and Mass Transfer*

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Review Convection Basics

- $\dot{Q}_{conv} = hA_{surface}(T_{surface} - T_{fluid})$ is heat transfer from surface to fluid
- $\dot{Q}_{conv} = hA_{surface}(T_{fluid} - T_{surface})$ is heat transfer from fluid to surface
- Physical heat transfer is opposite to assumed direction if $\dot{Q}_{conv}$ is negative
- Find h values from fluid and flow properties using empirical and theoretical results

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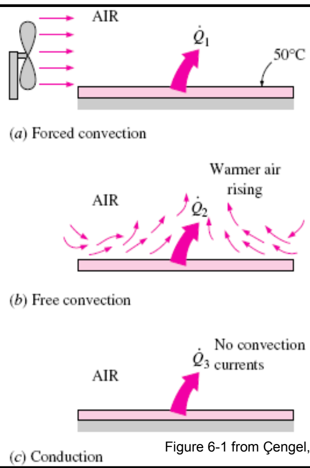

Mechanism

- Free convection has no imposed flow field
- Forced convection does
  - may come from motion of body
- Conduction only if no fluid motion

Figure 6-1 from Çengel, *Heat and Mass Transfer* 9

Flow Properties

- Moving fluid velocity components in x, y, and z directions are u, v, and w
- Fluids have shear stress, τ, that is proportional to velocity gradient and a property called the viscosity, μ
- For a simple flow in the x and y direction

$$\tau = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \approx \mu\frac{\partial u}{\partial y}$$

- Typically $\partial v/\partial x$ is negligible

California State University Northridge Figure 6-4 from Çengel, *Heat and Mass Transfer* 10

Viscosity Dimensions

- Dimensions of shear stress, τ, are force divided by area or $MLT^{-2}/L^2 = ML^{-1}T^{-2}$
- Dimensions of velocity gradient $\partial u/\partial y$ are L/T divided by T or T^{-1}
- Viscosity: $\mu = \tau / (\partial u/\partial y)$
- Dimensions of μ: $= ML^{-1}T^{-2} / T^{-1} = ML^{-1}T^{-1} = FTL^{-2}$ (F = force dimensions = MLT^{-2})
- Units for viscosity are kg/m·s = N·s/m^2 or $lb_f \cdot s/ft^2 = 32.174\ lb_m/ft \cdot s$

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Dynamic and Kinematic

- Viscosity, μ, defined previously is called the "dynamic" viscosity
- Define $\nu = \mu/\rho$, a common combination of properties, as "kinematic" viscosity
- Dimensions of ν are $(ML^{-1}T^{-1}) / (ML^{-3}) = L^2/T$ (same as those of $\alpha = k/\rho c_p$)
- Typical units are m^2/s or ft^2/s
- Can find ν in some property tables

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Effect of Viscosity

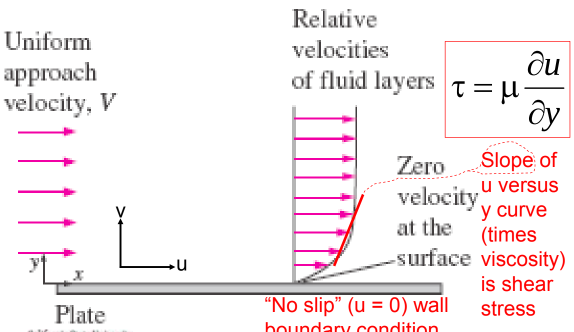

Figure 6-4 from Çengel, *Heat and Mass Transfer*

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Analysis at Walls

- At u = 0 wall heat transfer is all by conduction so: $\dot{q}_{wall} = -k_{fluid}(\partial T/\partial y)_{wall}$
  - This is basis for theoretical and computational analyses of convection
  - Have local, h_x, and average, h_{avg}, values

$$h_{avg} = \bar{h} = \frac{1}{L}\int_0^L h_x dx$$

- Wall shear stress: $\tau_{wall} = \mu(\partial u/\partial y)_{wall}$

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Flow Classifications

- Forced versus free
- Internal (as in pipes) versus external (as around aircraft)
  - Entry regions in pipes *vs*. fully-developed
- Unsteady (changing with time) versus unsteady (not changing with time)
- Laminar versus turbulent
- Compressible versus incompressible
- Inviscid flow regions (μ not important)
- One-, two- or three-dimensional

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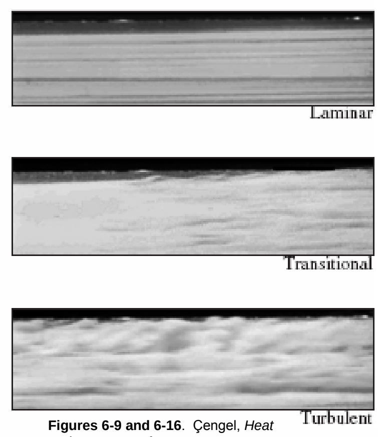

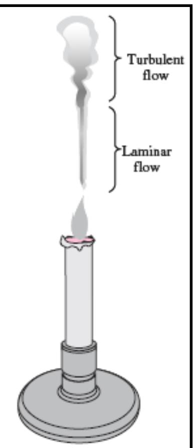

Figures 6-9 and 6-16. Çengel, *Heat and Mass Transfer*

Laminar and Turbulent Flows

- In laminar flows adjacent fluid layers remain in smooth contact
- Turbulent flow (much more common) is characterized by fluctuations in the flow
- Some flows can start as laminar then transition to turbulent
- Determination of laminar or turbulent flows is based on Reynolds number (forced) and Grashof number (free)

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Compressible Flows

- An incompressible flow may or may not be a constant density flow
  - In fluid mechanics an incompressible flow is one in which the changes in pressure do not significantly affect the density
    - Flows with large density changes due to temperature may be incompressible flows
- Incompressible flows have Mach numbers, Ma = V / a, less than 0.3 (a = sound speed)

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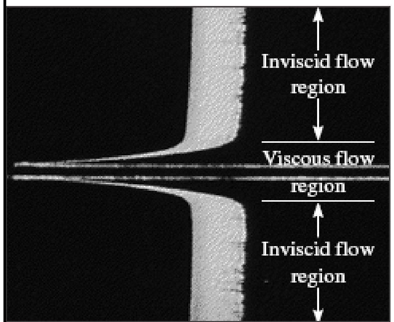
Inviscid Flow Regions



$$\tau = \mu \frac{\partial u}{\partial y}$$

- No flow is truly inviscid, (μ = 0) but regions of the flow may have $\partial u/\partial y \approx 0$ so $\tau \approx 0$

Figure 6-7 from Çengel, *Heat and Mass Transfer*

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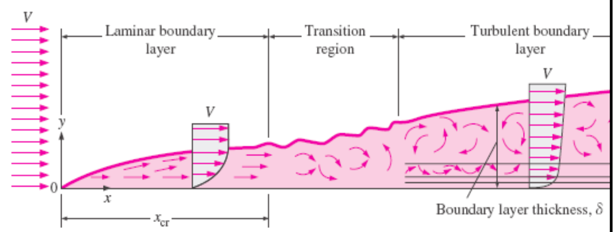
Boundary Layer



- Region near wall with sharp gradients
  - Thickness, δ, usually very thin compared to overall dimension in y direction

Figure 6-12 from Çengel, *Heat and Mass Transfer*

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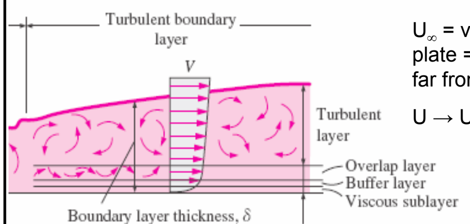
Boundary Layer



U_∞ = velocity before plate = velocity very far from plate

$U \to U_\infty$ as $y \to \infty$

- Boundary layer thickness, δ, defined as distance to point where velocity = $0.99U_\infty$

Figure 6-12 from Çengel, *Heat and Mass Transfer*

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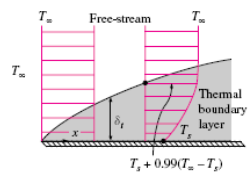
Thermal Boundary Layer

- Thin region near solid surface in which most of temperature change occurs



- Thermal boundary layer thickness may be less than, greater than or equal to that of the momentum boundary layer

Figure 6-15. Çengel, *Heat and Mass Transfer*

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Dimensionless Parameters

- Recall transient conduction analyses
  - Used Fourier number, τ, = $\alpha t/L_c^2$, Biot number, Bi = hL_c/k, and dimensionless distance x/L or r/R
    - L_c is characteristic length, L, R, D, etc.
- Found these by analysis of differential equation for conduction
- Allowed effect of several variables to be expressed in terms of a smaller number of dimensionless parameters

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Dimensionless Convection

- Nusselt number, Nu = hL_c/k_{fluid}
  - Different from Bi = hL_c/k_{solid}
- Reynolds number, Re = $\rho VL_c/\mu = VL_c/\nu$
- Prandtl number Pr = $\mu c_p/k$ (in tables)
- Grashof number, Gr = $\beta g \Delta T/\nu^2$
  - g = gravity, β = expansion coefficient = $-(1/\rho)(\partial\rho/\partial T)_p$, and $\Delta T = | T_{wall} - T_\infty |$
- Peclet, Pe = RePr; Rayleigh, Ra = GrPe

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Characteristic Length

- Can use length as a subscript on dimensionless numbers to show correct length to use in a problem
  - $Re_D = \rho VD/\mu$, $Re_x = \rho Vx/\mu$, $Re_L = \rho VL/\mu$
  - $Nu_D = hD/k$, $Nu_x = hx/k$, $Nu_L = hL/k$
  - $Gr_D = \rho^2\beta g\Delta TD^3/\mu^2$, $Gr_x = \rho^2\beta g\Delta Tx^3/\mu^2$, $Gr_L = \rho^2\beta g\Delta TL^3/\mu^2$
- Use not necessary if meaning is clear

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Dimensionless Example

- Heat transfer for turbulent flow in smooth tubes is given by the equation $Nu_D = 0.023\ Re_D^{0.8}Pr^n$ where n = 0.4 for heating and 0.3 for cooling
- Compute h for water if $T_{wall} = 80^oC$, $T_{fluid} = 40^oC$, D = 0.1 m, and V = 3 m/s
  - Evaluate properties at mean ("film") temperature of 60^oC
  - Use Table A-9, page 854, for properties

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Dimensionless Example II

- $Nu_D = 0.023\ Re_D^{0.8}Pr^{0.4}$ (heating because $T_{wall} = 80^oC$ and $T_{fluid} = 40^oC$), D = 0.1 m, and V = 3 m/s
- At mean temperature of 60^oC, $\rho = 983.3$ kg/m^3, k = 0.654 W/m·K, $\mu = 4.67\text{x}10^{-4}$ kg/m·s, Pr = 2.99

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{\frac{983.3\ kg}{m^3}\frac{3\ m}{s}(0.1\ m)}{\frac{4.67x10^{-4}\ kg}{m\cdot s}} = 6.32x10^5$$

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Dimensionless Example III

- $Nu_D = 0.023\ Re_D^{0.8}Pr^{0.4}$, D = 0.1 m, V = 3 m/s, $\rho = 983.3$ kg/m^3, k = 0.654 W/m·K, $\mu = 4.67\text{x}10^{-4}$, kg/m·s, Pr = 2.99, $Re_D = 6.32\text{x}10^5$

$$Nu_D = 0.023\,\mathrm{Re}_D^{0.8}\,\mathrm{Pr}^{0.4} = 0.023\left(6.32x10^5\right)^{0.8}(2.99)^{0.4} = 1557$$

$$h = \frac{kNu_D}{D} = \frac{\frac{0.654\,W}{m\cdot K}(1557)}{0.1\,m} = 1.02x10^4\ \frac{W}{m^2\cdot K}$$

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Governing Equations

- Fluid mechanics involves non-linear partial-differential equations
- Analytical solutions available only for simplest geometries
- Computational fluid dynamics is used to solve problems in convective heat transfer for complex geometries
- Text derives differential equations for simple case of boundary layers

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2D Boundary Layer Equations

- Continuity (mass conservation) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$
- Momentum conservation $\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$
- Energy conservation $\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$
- Steady, constant property, negligible dissipation of shear stress

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Dimensionless Forms

- Can convert boundary-layer equations into dimensionless forms as with unsteady heat conduction
- Define dimensionless quantities: $\xi = x/L_c$, $\eta = y/L_c$, $u' = u/U_\infty$, $v' = v/U_\infty$, $P' = P/\rho U_\infty^2$, and $\Theta = (T - T_\infty)/(T_s - T_\infty)$
- Substitute into equations on previous chart and carry out algebra to get results on next chart

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Dimensionless Forms II

- Continuity $$\frac{\partial u'}{\partial \xi} + \frac{\partial v'}{\partial \eta} = 0$$
- Momentum $$u'\frac{\partial u'}{\partial \xi} + v'\frac{\partial u'}{\partial \eta} = \frac{1}{\mathrm{Re}}\frac{\partial^2 u'}{\partial \eta^2} - \frac{\partial P'}{\partial \xi}$$
- Energy $$\left(u'\frac{\partial \Theta}{\partial \xi} + v'\frac{\partial \Theta}{\partial \eta}\right) = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left(\frac{\partial^2 \Theta}{\partial \xi^2} + \frac{\partial^2 \Theta}{\partial \eta^2}\right)$$

$$Nu = -\left(\frac{\partial \Theta}{\partial \eta}\right)_{\eta=0} = f(\mathrm{Re}, \mathrm{Pr}, \xi) \qquad \overline{Nu} = -\int_0^1 \left(\frac{\partial \Theta}{\partial \eta}\right)_{\eta=0} d\xi = f(\mathrm{Re}, \mathrm{Pr})$$

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Dimensionless Forms III

- The local Nusselt number, $Nu_x = hL_c/k$ depends on Re, Pr, and x/L_c
- The average Nusselt number, $Nu_{avg} = h_{avg}L_c/k$ depends on Re and Pr
- These relations, valid for the simple 2D case suggest correlations for empirical data
  - Expect $h_{average}$ = f(Re, Pr) for forced convection

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Skin Friction Coefficient, c_f

- Dimensionless shear stress $\tau' = \tau/\rho U_\infty^2$
- Wall (y = 0) shear stress = $\mu(\partial u/\partial y)_{y=0}$

$$c_{f_x} = 2\tau_w' = \frac{2\tau_w}{\rho U_\infty^2} = \frac{2}{\rho U_\infty^2}\left(\frac{\partial u}{\partial y}\right) = \frac{2}{\rho U_\infty^2}\mu\left(\frac{\partial (u' U_\infty)}{\partial (\eta L_c)}\right)_{y=0}$$

Local $$= \frac{2\mu}{\rho U_\infty L_c}\left(\frac{\partial u'}{\partial \eta}\right)_{\eta=0} = \frac{2}{\mathrm{Re}}\left(\frac{\partial u'}{\partial \eta}\right)_{\eta=0} = f(\mathrm{Re}, \xi)$$

Average $$c_f = \frac{1}{L}\int_{x=0}^{x=L} c_{f_x} dx = \int_{\xi=0}^{\xi=1} c_{f_x} d\xi = f(\mathrm{Re})$$

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Similarity

- For convection, the momentum and energy equations have similar forms
- Wall shear and heat transfer also have similar equations
  - Wall shear stress, $\tau_w = \mu(\partial u/\partial y)_{y=0}$
  - Wall heat transfer, $\dot{q}_{wall} = k(\partial T/\partial y)_{y=0}$
- We will find relations between the two in computations of h values

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Conclusions

- Theoretical equations for heat transfer coefficient are limited to very simple situations
- Dimensional analysis shows that h is related to flow properties through dimensionless variables
  - Nu = f(Re, Pr) for forced convection
  - Nu – f(Gr, Pr) for free (natural) convection
- Computations for h rely on equations between these dimensionless variables

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